On the Sobolev quotient of CR manifolds of 3D

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Abstract

We exhibit examples of compact three-dimensional CR manifolds of positive Webster class, *Rossi spheres*, for which the pseudo-hermitian mass as defined in [1] is negative, and for which the infimum of the CR-Sobolev quotient is not attained. To our knowledge, this is the first geometric context on smooth closed manifolds where this phenomenon arises, in striking contrast to the Riemannian case. This is joint work with Andrea Malchiodi and Paul Yang.

Keywords:CR manifold, Rossi sphere, pseudo-hermitian mass, CR-Sobolev quotient

References

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Poisson equation: estimates and applications

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In this talk, we intend to explain some estimates for the Green's function on complete manifolds admitting a weighted Poincaré inequality. The estimates are then applied to study the Poisson equation on such manifolds. Applications will also be mentioned. This is a joint work with Ovidiu Munteanu and Jiaping Wang.

Higher dimensional prescribed scalar curvature manifolds with horizon

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In this talk, we will construct higher dimensional asymptotically flat, time symmetric initial data with prescribed scalar curvature and with horizon. The idea comes from the Bartnik's quasi-spherical ansatz [1], Cabrera and Miao's observations on positive scalar curvature metrics on higher dimensional spheres [2], and Smith's construction on prescribed scalar curvature manifolds near the horizon [3].

References

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The sigularities of the network flow

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The network flow defines a geometric flow on an essentially singular geometric object. As other geometric flows, usually singularities will occur in finite time. We gives a description about the behaviors when the flow approaches the singular time. When using Huisken's monotonicity, the blow-up limits of a finite-time singularity should be a solution which shrinks self-similarly. This talk gives a survey of the known results about the classification of the self-similarly shrinking solutions and the open questions.

Keywords: Network flow, Singularities, Self-similar solution

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Spectral Stability of the $\bar{\partial}$-Neumann Laplacians

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We study spectral stability of the $\bar{\partial}$-Neumann Laplacian on a bounded domain in $\mathbb{C}^n$ when either the underlying domain or the operator is perturbed. This is a joint work with Siqi Fu.

When the perturbation of domain is measured by Hausdorff distance, we can establish upper semi-continuity properties for the variational eigenvalues of the $\bar{\partial}$-Neumann Laplacian on bounded pseudoconvex domains in $\mathbb{C}^n$, lower semi-continuity properties on pseudoconvex domains that satisfy property (P), and quantitative estimates on smooth bounded pseudoconvex domains of finite D'Angelo type in $\mathbb{C}^n$.

When the $\bar{\partial}$-Neumann Laplacian is perturbed by the Kohn-Nirenberg elliptic regularization which was obtained by adding a constant $t > 0$ times an elliptic operator to the $\bar{\partial}$-Neumann Laplacian. We will establish spectral stability of these operators as $t \to 0^+$ and show that the spectral stability of the Kohn-Nirenberg elliptic regularization requires less stringent condition on the boundary than the $\bar{\partial}$-Neumann Laplacian itself due to the coercive estimates. Indeed, we can establish a sharp quantitative estimate for the variational eigenvalues without the finite type assumption.

Keywords: $\bar{\partial}$-Neumann Laplacian, Spectrum; Stability, Pseudoconvex domain, Property (P), Finite type, Variational eigenvalue, Kohn-Nirenberg regularization.

Nonorientable Lagrangian surfaces in rational 4-manifolds

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The existence of Lagrangian submanifolds is an important problem in symplectic geometry. Most of the study focus on orientable cases. In this talk, we will give topological and homological constraints for nonorientable surfaces in symplectic rational 4-manifolds.

Keywords: Lagrangian surfaces, rational blow down, Lagrangian surgery

References

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